
Statistical Bioassay Theory, Design, and Computation

*A Graduate Textbook with Worked
Examples in R and Python*

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Dedication

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